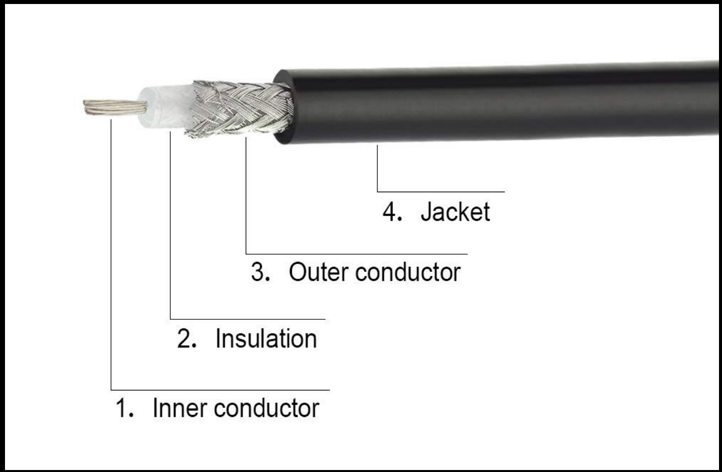
What is Characteristic Impedance?

(of Transmission lines)

You got yourself some!

- You bought a spool of RG8U 50Ω coaxial cable on Amazon
- You want to test it so you cut a 1' piece and strip the ends
- You connect an Ohmmeter and it reads….. $\infty\ \Omega$
- Something is wrong!
- Of course! the other end is open, so you short it and measure again
- The Ohmmeter now reads …. $0\ \Omega$
- Defective cable?

RG8U 50Ω Coax Cable



Hint

- Characteristic impedance is also known as the surge, or transient impedance

- The battery inside the Ohmmeter launched a wave into the 1’ coax, but light travels at approximately 300,000 km/s

  - Surge condition finished in the “blink of the eye”. too fast for the Ohmmeter to notice

You got yourself some! (2)

- Now, imagine you bought a spool of RG8U 50Ω coaxial cable on Amazon that contained an infinite length of RG8U.
  (It was on sale & free shipping for Prime members 😀)

- Strip one end and connect the Ohmmeter

- The Ohmmeter now reads ……50Ω

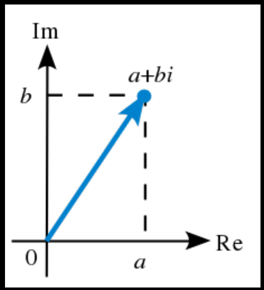
  - Why?

Why?

- Before we can answer why,
  we need to take a closer look at Impedance
- What is Impedance?
  - It's like Resistance, but includes Reactance
    - Measured in Ohms (Ω), but more complex
  - Ok, what the heck is Reactance?

Impedance – A Closer Look

- Impedance (Z) is a complex number!
  - Don't panic! The reason we use complex numbers is that we need to contend with both a magnitude and phase
  - Complex number have the form of



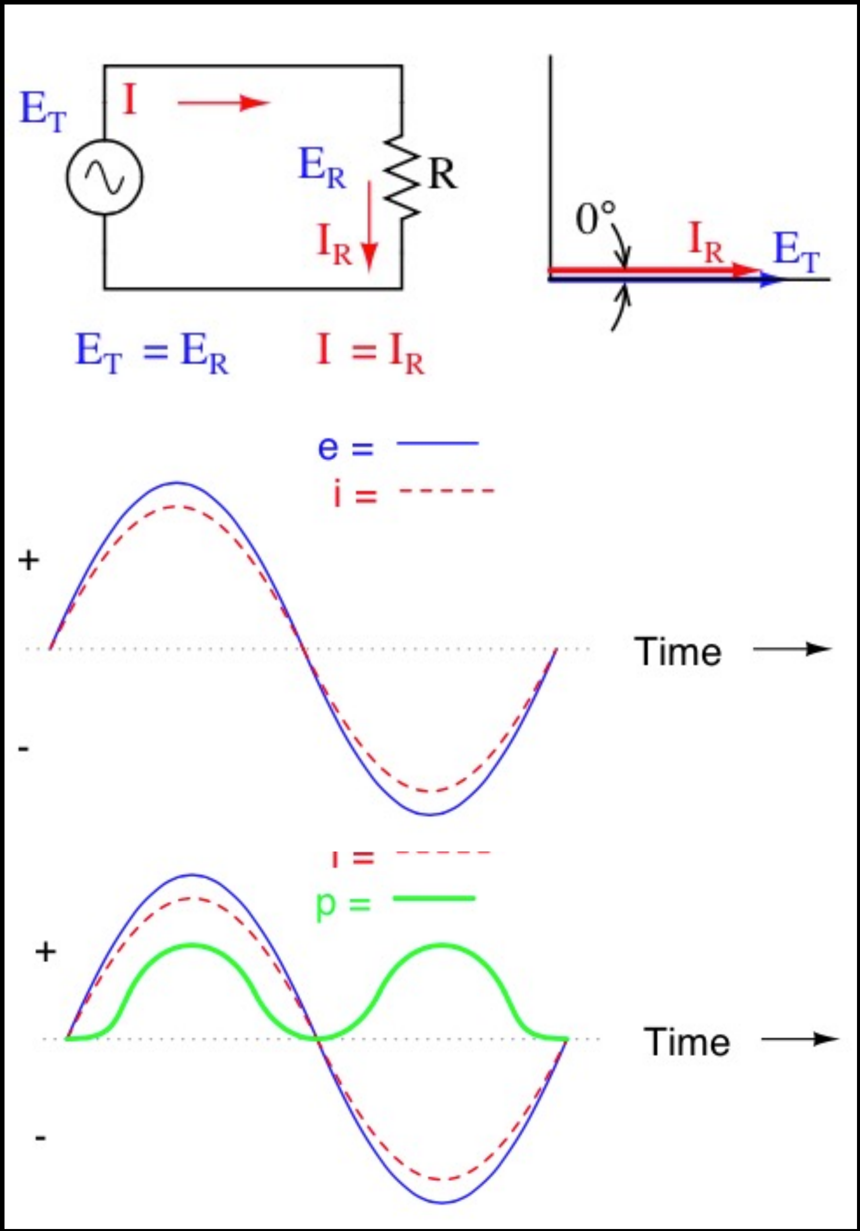
$$R_e \pm jI_m \text{ (where } j = \sqrt{-1}\text{)}$$

- Real Part: Resistive component, ie. Resistance (R)
- Imaginary Part: Reactive component, ie. Reactance (X)
  - Inductive, X_L or Capacitive, X_C

Resistance

In an ideal resistor (R):

- The Voltage (e) and Current (i) are always in phase with each other
- The impedance,(Z = R) is real & frequency independent
- The instantaneous Power (p = e x i) is always positive, meaning power is always dissipated (as heat)



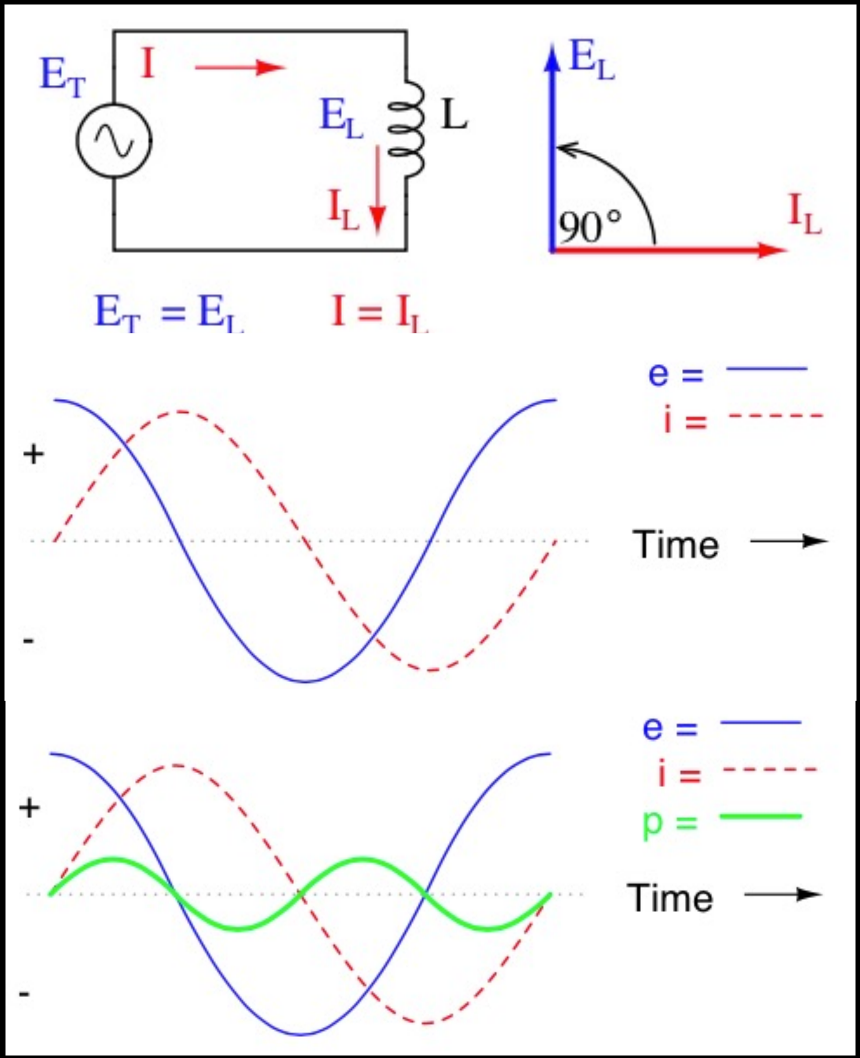
Inductance

In an ideal inductor (L):

- **The Voltage (e) wave leads the Current (i) wave by 90 degrees**
- **The impedance is imaginary and depends on the angular frequency (ω = 2πf)**

$$Z_L = j\omega L$$

- **The instantaneous Power (p = e X i) alternates sign meaning power is stored (as a magnetic field) and then returned to the circuit**



Capacitance

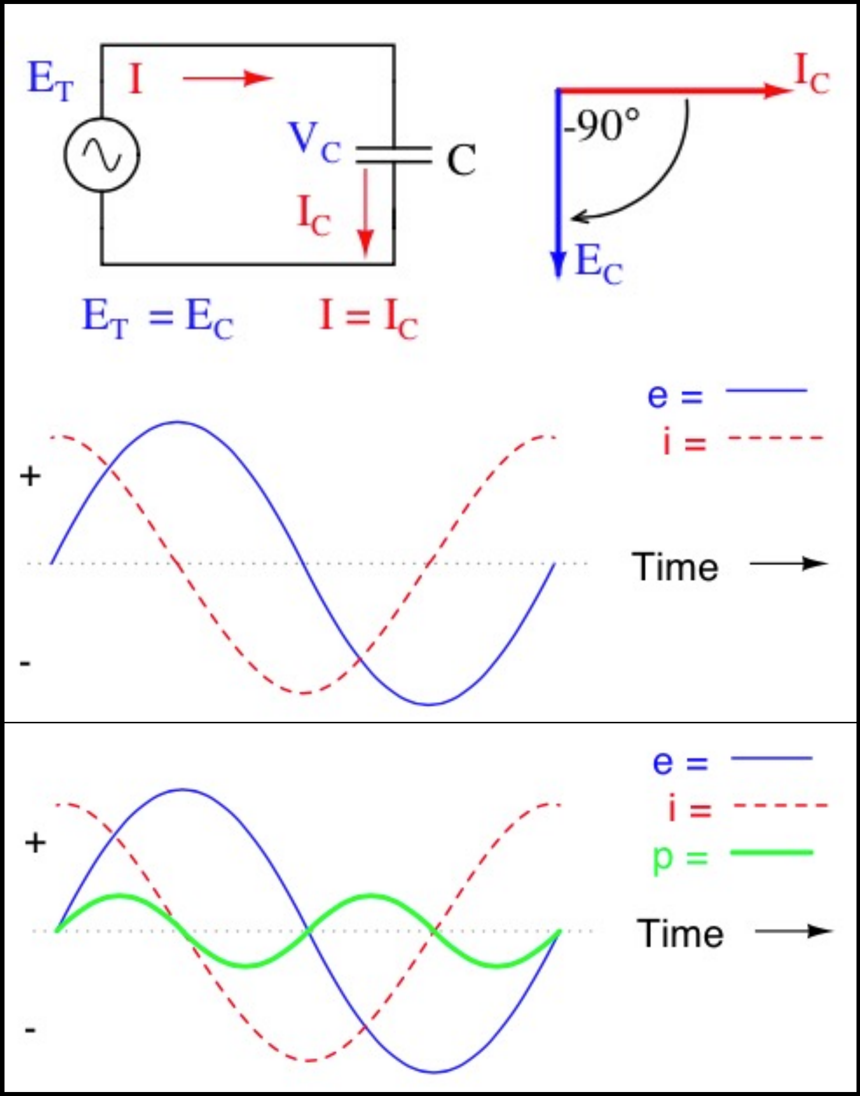
In an ideal Capacitor (C):

- **The Voltage (e) wave lags the Current (i) wave by 90 degrees**

- **The impedance is imaginary and depends on the angular frequency ($\omega = 2\pi f$)**

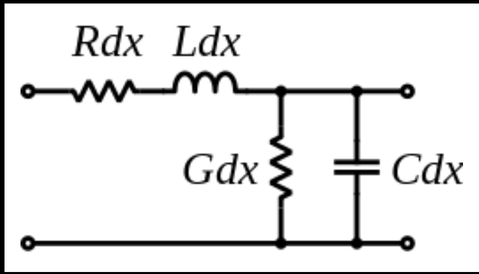
$$Z_C = -j\frac{1}{\omega C}$$

- **The instantaneous Power ($p = e \times i$) alternates sign meaning power is stored (as an electric field) and then returned to the circuit**



Distributed Transmission Line Model

Model as an infinite series of components, each representing an infinitesimally short segment of the transmission line

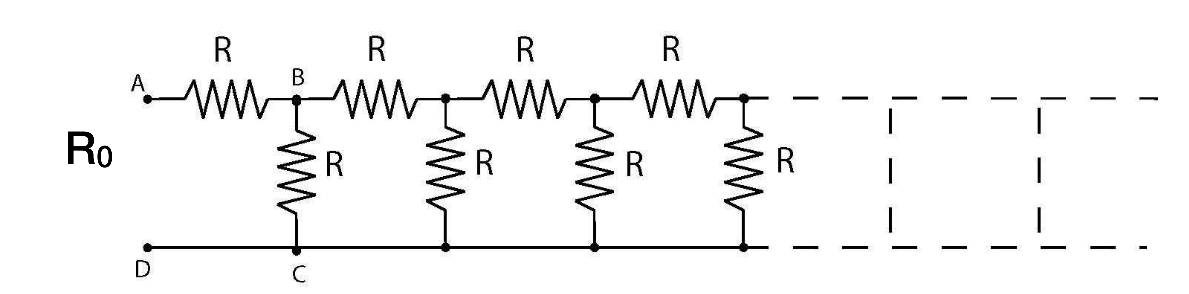


- The resistance Rdx of the conductors is represented by a series resistor (expressed in Ohms per unit length)
- The inductance Ldx, due to the magnetic field around the wires is represented by a series inductor (in Henries per unit length)
- The capacitance Cdx between the two conductors is represented by a shunt capacitor (in farads per unit length)
- The conductance Gdx of the dielectric material separating the two conductors is represented by a shunt resistor between the signal wire and the return wire (in Siemens per unit length, or if you are old like me, in mhos)

Conductance: G
G = 1/R

Resistor Ladder Example

Consider an infinite ladder of resistors of value R



What is the resistance of the ladder (R_0)?

Note that $R_0 = R + R \,||\, R_0$

Simply using the formula for resistors in parallel and solve for R_0

The answer is:

$$R_0 = \frac{(1+\sqrt{5})R}{2}$$

Serendipity

Architects and artists may recognize

$$\frac{(1+\sqrt{5})}{2} = 1.618033988....$$

As "The Golden Ratio"

Transmission Line

- We can solve for Z_0, just like we did for the resistor ladder

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

- Z_0 varies with frequency (ω)
- What happens if R and G are so small that they can be ignored?
  - $R = 0$; $G = 0$
- Lossless line
  - Z_0 is a constant and independent of frequency

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

When is it a Transmission Line?

- When do we need to consider a connection as a Transmission Line, rather than just a connecting wire?
  - When the signal frequencies are high enough that their wavelengths are about 10% of the connection distance
    - That is, when the line length is $\geq$ ~0.1λ
      - Recall that $\lambda = c/f$, where c is the speed of light

Z_0 of Coaxial Line

You can also calculate Z_0 for lossless lines from empirical formulae

$$\boxed{Z_0 = \frac{138}{\sqrt{k}} \log \frac{d_1}{d_2}}$$

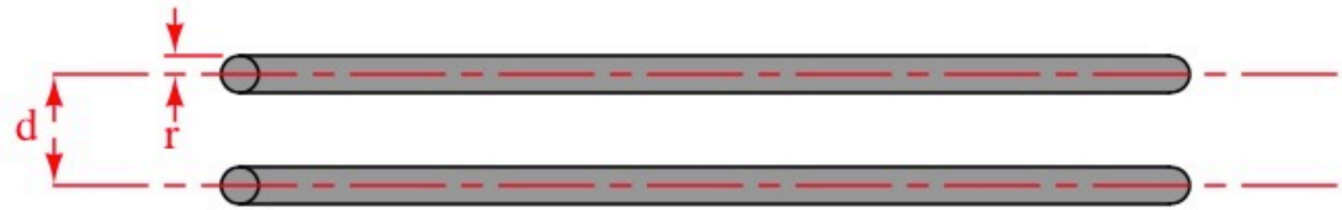
Where,

Z_0 = Characteristic impedance of line
d_1 = Inside diameter of outer conductor
d_2 = Outside diameter of inner conductor
k = Relative permittivity of insulation between conductors

- **Relative permittivity (aka Dielectric Constant), is a measure of how easily a material can become polarized by an electric field**
  - **It also affects the propagation speed and wavelength of the waves**

$$c' = \frac{c}{\sqrt{K}},$$

Z_0 of Parallel Wires



$$Z_0 = \frac{276}{\sqrt{k}} \log \frac{d}{r}$$

Where,

Z_0 = Characteristic impedance of line
d = Distance between conductor centers
r = Conductor radius
k = Relative permittivity of insulation between conductors

Dielectric Constant of Some Materials

Material	Relative Permittivity
Vacuum	1.0000
Air	1.0006
PTFE, FEP (Teflon)	2.0
Polypropylene	2.20 to 2.28
Polystyrene	2.4 to 3.2
Wood (Oak)	3.3
Bakelite	3.5 to 6.0
Wood (Maple)	4.4
Glass	4.9 to 7.5
Wood (Birch)	5.2
Glass-Bonded Mica	6.3 to 9.3
Porcelain, Steatite	6.5

END